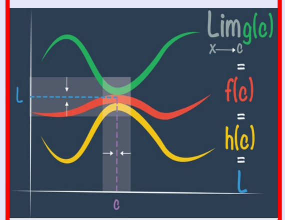
Math 261

Fall 2022

Lecture 31



Given $f(x) = -x^4 + 2x^2 + 2$

Polynomial Function $\rightarrow$ Cont. everywhere

Domain $(-\infty, \infty)$

$f(-x) = -(-x)^4 + 2(-x)^2 + 2 = -x^4 + 2x^2 + 2 = f(x)$

$f(-x) = f(x)$ $\rightarrow$ even Function $\rightarrow$ symmetric with respect to Y-axis.

$f(0) = 2$

$f'(x) = -4x^3 + 4x = -4x(x^2-1) = -4x(x+1)(x-1)$

C.N. $\rightarrow$ $f'(x) = 0$ or undefined $\rightarrow$ $x = 0, -1, 1$

$-4x(x+1)(x-1) = 0$

C.P. $(0, f(0)) = (0, 2)$

$(1, f(1)), (-1, f(-1)) \rightarrow$ $(1, 3)$, $(-1, 3)$

$f''(x) = -12x^2 + 4 = -4(3x^2 - 1)$

P.I.P. $\rightarrow$ $f''(x) = 0$ or undefined

$-4(3x^2-1) = 0 \longrightarrow x = \pm \frac{1}{\sqrt{3}}$

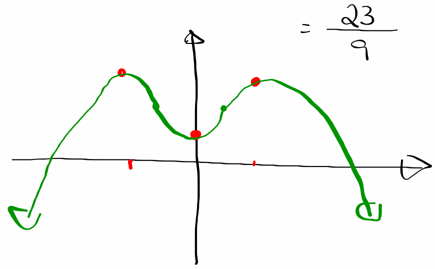
x	$-\infty$		-1		$\frac{-1}{\sqrt{3}}$		0		$\frac{1}{\sqrt{3}}$		1		∞
$f'(x)$		$+$	•	$-$		$-$	•	$+$		$+$	•	$-$	
$f''(x)$		$-$		$-$	•	$+$		$+$	•	$-$		$-$	
$f(x)$		↗	Max	↘	I.P.	↘	Min	↗	I.P.	↗	Max	↘	

$f(1)=3$, $f(-1)=3$

$$f\left(\frac{1}{\sqrt{3}}\right) = -\left(\frac{1}{\sqrt{3}}\right)^4 + 2\left(\frac{1}{\sqrt{3}}\right)^2 + 2 = \frac{-1}{9} + \frac{2}{3} + 2 = \frac{-1}{9} + \frac{6}{9} + \frac{18}{9} = \frac{23}{9}$$

$f\left(\frac{1}{\sqrt{3}}\right) = \frac{23}{9}$

$f\left(\frac{-1}{\sqrt{3}}\right) = \frac{23}{9}$



Given $f(x) = \frac{x}{x-1}$

Domain: $x \neq 1$ $(-\infty,1)\cup(1,\infty)$

$\lim_{x\to\pm\infty} f(x) = 1$

$\boxed{\text{V.A. } x=1}$ $\boxed{\text{H.A. } y=1}$

$f(0) = \frac{0}{-1} = 0$

x-Int $(0,0)$, Y-Int $(0,0)$

$f(-x) = \frac{-x}{-x-1} = \frac{-x}{-(x+1)} = \frac{x}{x+1}$ neither

Find $f'(x)$

$$f(x) = \frac{x-1+1}{x-1} = \frac{x-1}{x-1} + \frac{1}{x-1} = 1+(x-1)^{-1}$$

$$f'(x) = -1(x-1)^{-2}\cdot 1 = -(x-1)^{-2} = \frac{-1}{(x-1)^2}$$

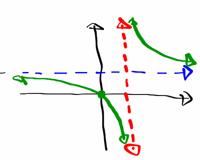
$f'(x) \neq 0$, $f'(x)$ undefined at $x=1$, $f'(x)<0$

$$f''(x) = -\cdot -2(x-1)^{-3}\cdot 1 = \frac{2}{(x-1)^3}$$

$f''(x) \neq 0$

$f''(x)$ is und. at $x=1$

x	$-\infty$		1		∞
$f'(x)$		$-$	⋮	$-$	
$f''(x)$		$-$	⋮	$+$	



Given $f(x)=\frac{x}{\sqrt{x^2+1}}$

Domain $(-\infty,\infty)$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{\frac{x}{x}}{\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}=1$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{\frac{x}{x}}{-\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}}=-1$$

$$f(-x)=\frac{-x}{\sqrt{(-x)^2+1}}=\frac{-x}{\sqrt{x^2+1}}$$

$$=-f(x)$$

odd function

Symmetry → Origin

$$f'(x)=\frac{1}{(x^2+1)^{3/2}}>0$$

wolframalpha.com

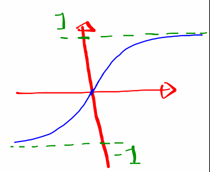
$$f'(x)=(x^2+1)^{-3/2}\qquad f''(x)=-\frac{3}{2}(x^2+1)^{-5/2}\cdot 2x$$

$$f''(x)=\frac{-3x}{(x^2+1)^{5/2}}$$

P.I.P. at $x=0$

$f(0)=0$

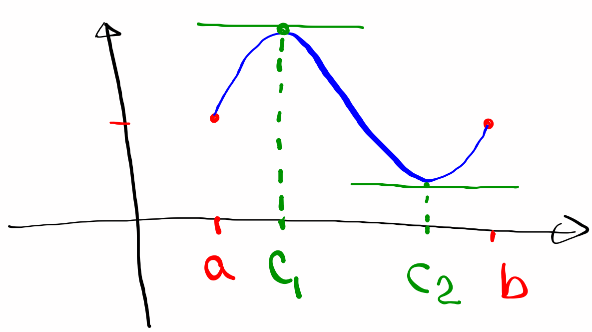
x	$-\infty$	0	∞
$f'(x)$	+		+
$f''(x)$	+		−
$f(x)$	↗		↗



Suppose $f(x)$ is cont. on $[a,b]$ and it is differentiable on (a,b) and $f(a)=f(b)$, then

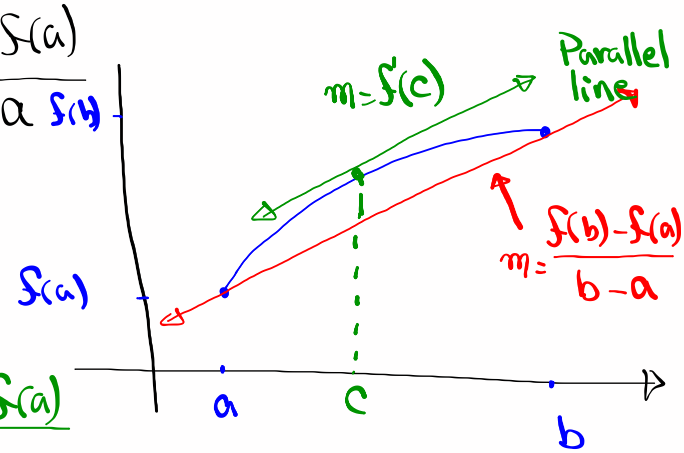
There is at least a number c in (a,b) such that $\boxed{f'(c)=0}$

Rolle's Theorem



Suppose $f(x)$ is Cont. on $[a,b]$ and is diff. on (a,b), then there is at least a number c in (a,b) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

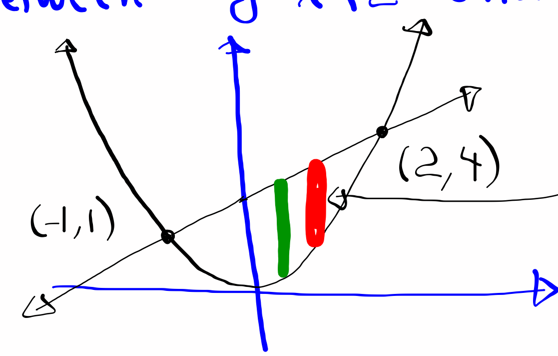


Parallel lines have Same slope

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Mean-Value Theorem

What is the maximum vertical distance between $y = x+2$ and $y = x^2$ for $-1 \le x \le 2$



$D = \text{Top} - \text{Bottom}$

$= x+2 - x^2$

$D(x) = -x^2 + x + 2$

$D'(x) = -2x + 1$

$D''(x) = -2$

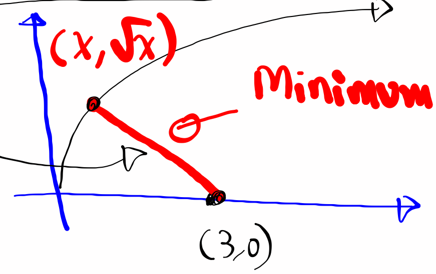
Max

$D'(x) = 0 \rightarrow -2x+1 = 0 \quad x = \frac{1}{2}$

$$D\left(\tfrac{1}{2}\right) = -\left(\tfrac{1}{2}\right)^2 + \tfrac{1}{2} + 2 = -\tfrac{1}{4} + \tfrac{1}{2} + 2 = \tfrac{1}{4} + 2 = \boxed{2.25}$$

find $\boxed{\text{a point}}$ on $\boxed{y=\sqrt{x} \text{ graph}}$ that is the closest to $(3,0)$

$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$



$D = \sqrt{(x-3)^2+(\sqrt{x}-0)^2}$

$D = \sqrt{(x-3)^2+x} = \sqrt{x^2-6x+9+x} = \sqrt{x^2-5x+9}$

minimize $d = x^2-5x+9$

$d' = 2x-5$

$d'' = 2$

$2x-5=0$

$x = 5/2 = 2.5$

At $(2.5, \sqrt{2.5})$, we have a minimum distance to $(3,0)$